CLIL Module Plan

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School	Liceo Scientifico "Leonardo da Vinci" Trento				
School Grade	○ Primary		○ Middle		◉ High
School Year	○ 1	○ 2	○ 3	○ 4	◉ 5
Subject	Matematica	**Topic**	An introduction to integrals		
CLIL Language	◉ English		○ Deutsch		

Personal and social-cultural preconditions of all people involved	The scientific high school “Leonardo da Vinci” is one of the historical “Liceo” of the Province of Trento. Nowadays the school proposes two curricula, the ordinary scientific curriculum and the applied sciences scientific curriculum. The class consists of 20 students. There are no SEN students or students of foreign origin. The large part of the lessons of the present CLIL module takes place in the classroom. This is equipped with a PC, an interactive whiteboard (IWB), a large blackboard, and a projector. A few activities are carried out in the ICT lab, where each student can use a PC and in particular the software GeoGebra. The academic performance of the class for what concerns the scientific subjects is average. The behaviour of the students is polite although the level of participation is not always high. The majority of the students are highly motivated and willing to learn new concepts, however there are elements of the class that do not have a specific interest in the subject or feel the foreign language as a barrier. The teachers who will carry out the CLIL Module (“T” in the Module Plan) are Giovanni Lombardi and Ilaria Geat. They will be the main teachers in the class for this CLIL module. Giovanni Lombardi teaches Mathematics and Physics and Ilaria Geat teaches Physics in various classes of the school. They both have a C1 English level certification. The Mathematics teacher in this class does not have a sufficient level of spoken English to actively participate in the lessons. However, she will listen (she understands English quite well) to the students, observe them, encourage them to participate more actively when needed. All the students are Italian mother tongue, and their average CEFR level is B2, but a few already have a C1 certification. The students already have some CLIL experience as they have followed CLIL lessons for a total of 60 hours during the two previous school years.

Students' prior knowledge, skills, competencies	Subject	Language
	Students already have studied limits and they can work with them. They know the definition of a continuous function and the main theorems regarding continuous functions. Students have already studied derivatives and they can calculate them. In particular they can calculate the derivative of a product of functions and of a composition of functions. They also have some experience in using the software GeoGebra.	Students have good reading and writing skills and adequate communication skills. They can interact both with the teachers and with their fellow students in English. Since they have followed another CLIL Module in Mathematics during the previous year, they already know all the basic mathematical terms. Other specific terms will be introduced during the lessons.

Timetable fit	◉ Module	Length 20 hours, 1 Unit, 10 Lessons

Description of teaching and learning strategies	• The learning and teaching objectives are disciplinary (and interdisciplinary) specific, transversal and communicative. The lessons have been designed to encourage the development of problem solving skills, critical thinking, creative thoughts and ideas, collaboration, communication and time managing. • The methodological approaches will be various, in order to meet different learning styles and to promote the development of different skills: interactive lessons, group work (especially when the task is complex), pair work, individual work, TPS (Think, Pair, Share), cooperative learning. During the “student-centred” activities the teachers will act as facilitators and guides. • Interaction and communication are promoted as much as possible by the teachers, by asking questions and inviting the students to comment or express their ideas, and by group or pair work. During these activities, the teachers circulate and model language, concepts and cognition. A quiz and two short competitions at the end of two lessons contribute to promote interaction. • The software GeoGebra is employed in many activities (mostly based on pair/group work) throughout the module as an ICT learning tool. The students are often encouraged to use this software as a means to explore new problems. In addition a variety of online resources (including online lessons and practice material) from Khan Academy is used. • The teachers provide different materials to support content and language scaffolding, and to consolidate learning, e.g. worksheets, extra exercises, and homework. • During most activities a formative assessment by the teachers is provided and peer- or self-evaluation is encouraged, while at the end of the Module a summative assessment is provided.

Overall Module Plan

Unit: 1 An introduction to integrals **Unit length:** 20 hours	**Lesson 1** Approximating the area of the circle
	Lesson 2 Approximation of the area under the graph of a function using rectangles
	Lesson 3 Integral of a function and area under the graph of the function
	Lesson 4 Integrals – properties
	Lesson 5 The integral function
	Lesson 6 Integrals – the fundamental theorem of calculus
	Lesson 7 Methods of integration
	Lesson 8 Integrals – Problem solving challenge
	Lesson 9 Integrals – physics applications + exercises in preparation to the unit test
	Lesson 10 Unit test

CLIL Lesson Plan

Unit number	1	**Lesson number**	1	**Title**	Approximating the area of the circle

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**
1	10'	Having an overview of the activities that will be carried out during this module.	a) T introduces and explains the aims, topic and objectives of the CLIL Module “An introduction to integrals”. b) T introduces the topic and learning outcomes of this first lesson. c) Ss take notes.	**Skills** L S R W **Key vocabulary** Plan, area, integral, function, theorem **Communicative structures** “We are going to study…” “Our objective is…” “If you have any questions…”	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work	The lesson plan (file LP.pdf) is shown on the interactive whiteboard (IWB).	

2	10'	a) Making hypotheses. b) Improving one's own problem solving skills. c) Modelisation.	a) T sets the goal for the activity: finding the formula that describes the area of a generic isosceles triangle inscribed in a circle of radius R. b) Ss make hypotheses c) Ss give opinions/comments about peers' hypotheses d) the area is written in a "rough" form.	**Skills** L S R W **Key vocabulary** circle, isosceles triangle, angle, chord, apex, legs, vertex angle, base angles, sine, cosine, tangent **Communicative structures** "Let's call the vertex angle α" "The base of the triangle corresponds to the chord..." "We could write the length of the base as..." "The height results..." "I agree/disagree"	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T writes on the blackboard the main points of the hypotheses made by Ss. Ss take notes on their personal notebooks.	Formative: T assesses the insight of the hypothesis made by Ss. T informally assesses the language used to formulate the hypotheses.

3	15'	a) Practicing the use of trigonometry relations. b) Manipulating/simplifying a mathematical expression. c) Comparing own results to peers'. d) Cooperating.	a) T asks the students to simplify the function found in the previous activity using their knowledge of trigonometry. b) Ss work individually for 5 minutes to simplify the formula for the area found in the previous activity. c) After 5 minutes expire Ss form pairs. d) Ss compare their results to those of their pair-mates.	**Skills** L S R W **Key vocabulary** circle, isosceles triangle, angle, chord, apex, legs, vertex angle, base angles, sine, cosine, tangent **Communicative structures** "My result is... "I agree/disagree" "I believe I/you made a mistake:..." Sentence structures related to mathematical relations.	☐ Whole class ☐ Group work ☒ Pair work ☒ Individual work	Ss exploit the calculations in their personal notebooks.	Self-assessment: Ss compare their version of the solution to that found by their pair-mates.

4	15'	a) Employing the knowledge about derivatives. b) Solving a problem involving the maximisation of a function. c) Improving one's own problem solving skills. d) Presenting results to peers. e) Comparing own results with those of peers. Giving opinions/comments.	a) T asks Ss to determine the triangle of maximum area inscribed in a circle of radius R. b) Ss maximise the function describing the area of the generic isosceles triangle inscribed in the circle. c) After 10 minutes the solution is discussed with the whole class.	**Skills** L S R W **Key vocabulary** circle, isosceles triangle, vertex angle, area, function, variable, derivative, maximum, sine, cosine, tangent **Communicative structures** “We try to maximise the function with respect to the variable...” “We set the derivative with respect to... Equal to...” “Can you explain...?”	☑ Whole class ☐ Group work ☐ Pair work ☑ Individual work	Ss solve the problem on their personal notebooks. At the end of the activity the problem is solved at the blackboard for the whole class.	During the activity T goes around the class evaluating the level of participation and comprehension of Ss (asking targeted questions if necessary).

5	10'	a) Making hypotheses. b) Improving one's own problem solving skills. c) Modelisation.	a) T sets the goal for the activity: to design a strategy to calculate the area of a regular n-sided polygon inscribed in a circle of radius R. b) Ss make hypotheses. c) Ss give opinions/comments about peers' hypotheses. d) T summarises at the blackboard the main ideas emerged during the discussion.	**Skills** L S R W **Key vocabulary** circle, regular polygon, angle, chord, length, sine, cosine, tangent **Communicative structures** "The side of the polygon corresponds to the chord..." "We could divide the polygon in isosceles triangles and..." "The height results..." "I agree/disagree"	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T writes on the blackboard the main points of the hypotheses made by Ss. Ss take notes on their personal notebooks.	Formative: T assesses the insight of the hypothesis made by Ss. T informally assesses the language used to formulate the hypotheses.

6	15'	a) Employing the knowledge acquired during the previous activities. b) Practicing the use of trigonometry relations. c) Manipulating/simplifying a mathematical expression. d) Comparing own results to peers’. e) Cooperating.	a) T asks Ss to calculate the area of a regular n-sided polygon inscribed in a circle of radius R following the strategy designed during the previous activity. b) Ss work individually for 10 minutes to obtain and simplify the formula for the area. c) After 5 minutes expire the solution is discussed with the whole class. d) The formula A_n= (n/2)*r^2*sin(2π/n) is found.	**Skills** L S R W **Key vocabulary** circle, regular polygon, angle, chord, length, sine, cosine, tangent **Communicative structures** “The side of the polygon corresponds to the chord...” “The area of the small triangles is...” “I agree/disagree” “We obtain the formula...”	☑ Whole class ☐ Group work ☐ Pair work ☑ Individual work	Ss exploit the calculations in their personal notebooks. T writes the solution on the blackboard.	Self-assessment: Ss compare their version of the solution to that found by their pair-mates.

7	15'	a) Employing the knowledge acquired during the previous activities. b) Practicing the use of trigonometry relations. c) Manipulating/simplifying a mathematical expression. d) Comparing own results to peers'. e) Cooperating.	a) T asks Ss to work in pairs to calculate the area of a regular n-sided polygon circumscribed about a circle of radius R. b) Ss work in pairs for 10 minutes to design a strategy and obtain the formula for the area. c) After 10 minutes expire the solution is discussed with the whole class.	**Skills** L S R W **Key vocabulary** circle, regular polygon, circumscribed about, angle, chord, length, sine, cosine, tangent **Communicative structures** "We can use a strategy similar to..." "The heigth of the small triangle corresponds to the radius" "The area of the small triangles is..." "I agree/disagree" "We obtain the formula..."	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	Ss exploit the calculations in their personal notebooks. T writes the solution on the blackboard.	During the activity T goes around the class assessing the level of participation and comprehension of Ss. Self-assessment: Ss compare their version of the solution to that found by their pair-mates.

8	10'	a) Employing the knowledge about limits. b) Giving opinions/comments. c) Critical thinking. d) Checking the validity of a result.	a) T leads the work of the class to calculate the limit for n going to infinity of the area of the regular n-sided polygon inscribed in/circumscribed about a circle of radius R. b) Ss share their results. c) Ss discuss the results. d) It is important that Ss realise that the result of the limit should be equal to the area of the circle.	**Skills** L S R W **Key vocabulary** circle, regular polygon, inscribed in, circumscribed about, angle, limit, infinity **Communicative structures** “The calculation of the limit yields...” “Do you think the result is right? Why?” “I agree/disagree”	▣ Whole class ☐ Group work ☐ Pair work ▣ Individual work	Ss exploit the calculations in their personal notebooks. T writes the solution on the blackboard.	Formative: T assesses the insight of the hypothesis made by Ss. T informally assesses the language used to formulate the hypotheses.

CLIL Lesson Plan

Unit number	1	**Lesson number**	2	**Title**	Approximation of the area under the graph of a function using rectangles

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**

1	15'	a) Translating a real life problem into a mathematical problem. b) Understanding that Mathematics can be useful not only to solve abstract exercises in books, but also to solve real life problems. c) Reading, understanding and interpreting a graph. c) Creative thinking. d) Discussion and communication.	a) T briefly introduces objective and learning outcomes of this lesson. b) T shows two graphs on the IWB (see U1_L2_ALL1) regarding the variation of the pollution level in two different places in Trento from the 19th September to the 19th October. c) T asks Ss the following question: “Which place was the most polluted in the given period?” d) T conducts the discussion, guiding Ss with some appropriate questions. e) In conclusion Ss should understand that, in order to answer the question, they need to determine the areas under the curves represented in the graphs and compare them. f) The problem is now: how to determine the area under a curve.	**Skills** L S R W **Key vocabulary** pollution, graph, area, curve, polygon **Communicative structures** “Today we’re going to discuss a real life problem…” “What do you need in order to answer the question?” “If I knew the area under that curve, then…” “But how can we find that area? This region is not a polygon…”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L2_ALL1.pdf T shows the file U1_L2_ALL1 on the IWB.	Formative: T models language and cognition.

2	25'	a) Improving problem solving skills. b) Creative thinking: thinking about an approximate solution for the problem. c) Using GeoGebra to explore/solve a problem. d) Using the spreadsheet in GeoGebra. e) Communication and discussion.	a) Ss are going to solve the problem “how to determine the area under a curve” at first with an easy curve (parabola of equation y=x^2). b) Ss should now think about the problem and suggest ideas for solving it. c) T conducts the discussion so that in the end they decide that the easiest way to solve the problem is approximating the unknown area with some rectangles. d) They are going to do this using GeoGebra. e) T shows Ss how to create a GeoGebra file (see U1_L2_ALL2) where the area under the parabola between one and two is rounded down by the area of five rectangles. f) Ss follow T and create the same file. g) Ss ask for clarifications about the creation of this file if needed.	**Skills** L S R W **Key vocabulary** to round up/down, approximation, rectangle, spreadsheet **Communicative structures** “Let’s try to solve the problem in a simple case: our curve is the parabola of equation…” “We could approximate the area under the curve using geometrical shapes with known area.” “Which geometrical shape would you choose?” “Excuse me, could you show me/us again how to draw…?”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L2_ALL2.zip	Formative: T models language and cognition.

3	15'	a) Using GeoGebra to explore/solve a problem. b) Using the spreadsheet in GeoGebra. c) Analyzing results. d) Cooperation. e) Communication.	a) T forms small groups and asks Ss to create a GeoGebra file similar to the previous. This time Ss should round up the unknown area with five rectangles, instead of rounding it down. b) Ss work in groups and create a file similar to U1_L2_ALL3. c) Then T asks Ss to compare the area they found with this approximation with the area they found in the previous activity. d) T circulates to facilitate.	**Skills** L S R W **Key vocabulary** to round up/down, spreadsheet, approximation **Communicative structures** “In your group create another file to round up…” “This time we should take these rectangles…” “The total area is greater than…”	☐ Whole class ☒ Group work ☐ Pair work ☐ Individual work	• U1_L2_ALL3.zip	Formative: T models language and cognition.

4	25'	a) Using GeoGebra to explore/solve a problem. b) Using the spreadsheet in GeoGebra. c) Analyzing and comparing results. d) Cooperation. e) Communication.	a) Now T asks Ss to repeat the previous tasks and improve them by rounding up and down the unknown area, using ten rectangles instead of five. b) Ss work in groups and create two other GeoGebra files, similar to U1_L2_ALL4. c) Ss are asked to compare the results with previous results. d) T circulates to facilitate.	**Skills** L S R W **Key vocabulary** to round up/down, spreadsheet, approximation **Communicative structures** “Now round it up and down as before using ten rectangles instead of five.” “We’ll get a better approximation, don’t you agree?” “This area is greater than… while the other is smaller than…”	☐ Whole class ☒ Group work ☐ Pair work ☐ Individual work	• U1_L2_ALL4.zip	Formative: T models language and cognition.

5	10'	a) Exploring a GeoGebra file. b) Understanding the basic idea of the integral. c) Generalizing results. d) Developing discussion.	a) T shares a GeoGebra file with Ss that generalizes what has just been done. Thanks to a slider it is possible to increase the number of rectangles in the interval [1;2] up to a hundred. b) Ss can now explore the file in groups. c) Ss observe the values of the “LowerSum” and the “UpperSum” (the areas of all the rectangles approximating the unknown area up and down respectively) as function of n (number of rectangles in the interval), take notes and discuss in groups.	**Skills** L S R W **Key vocabulary** slider, to round up/down, approximation **Communicative structures** “Look at the values of “LowerSum” and “UpperSum”: if n increases then….” “Set n equal to a hundred and let’s see what happens.” “If n is equal to five or ten we get the previous results…” “The difference between the two approximations decreases when n…”	☐ Whole class ☒ Group work ☐ Pair work ☐ Individual work	• U1_L2_ALL5.zip	

6	10'	a) Communication and discussion. b) Drawing conclusions and generalizing.	a) T asks Ss to share their results and ideas in plenary. b) Ss discuss in plenary. c) T sums up what Ss have just understood and generalizes: the greater the number of rectangles is, the more precise is the approximation.	**Skills** L S R W **Key vocabulary** approximation, to round up/down, slider **Communicative structures** “Now you can share your ideas with the rest of the class.” “What did you notice when using the slider?” “We noticed that if n increases the values of “LowerSum” and “UpperSum”...”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L2_ALL5.zip T sums up what Ss have just understood and generalizes on the blackboard.	Formative: T elicits language and cognition.

CLIL Lesson Plan

Unit number	1	**Lesson number**	3	**Title**	Integral of a function and area under the graph of the function

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**
1	30'	a) Understanding the definition of the integral of a function and its relation to the area under the graph of the function. b) Interaction and communication. c) Listening to others.	a) T introduces objective and learning outcomes of this lesson. b) T introduces the concept of the integral of a positive function, of a negative function and of a generic function and gives examples on the blackboard. c) While explaining, T tries to make Ss participate as much as possible, asking them questions and presenting the definition of integral as the (signed) area under the graph of the function. In this way the integral is presented as the mathematical tool that models the original	**Skills** L S R W **Key vocabulary** Positive/negative function, signed area, integral (from a to b), integral over the interval [a;b], limits of an integral.	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T writes examples and definitions on the blackboard. Ss take notes in their personal notebooks.	

			models the original problem of finding the area under a curve. d) Ss interact with T and ask questions if they have any doubts.	**Communicative structures** “Today we’re going to define…” “If the function is negative, then…” “Do you have any idea about how to define the integral of a function that changes its sign in the interval considered?” “I think we could divide the interval…”			
2	20'	a) Applying a known procedure to a new problem. b) Using GeoGebra as a tool for approximating areas. c) Cooperation and communication. d) Comparing own results to peers’.	a) T hands out one laptop per pairs and the worksheets for the first problem Ss are asked to solve. b) Ss solve this problem in pairs using GeoGebra. c) T circulates to facilitate. d) After 15 minutes, T asks Ss to share their results in plenary.	**Skills** L S R W **Key vocabulary** area, approximation, rectangles, round down, round up **Communicative structures** “I think we could…” “We can use a strategy similar to…” “We got the following result…”	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	• U1_L3_ALL1.pdf	Formative: T models language and concepts. Peer and self-assessment.

3	25'	a) Improving one’s own problem solving skills. b) Using GeoGebra as a tool to solve a problem. c) Exploring new functions in GeoGebra and using them to obtain a specific result. d) Critical thinking. e) Cooperation and communication. f) Comparing own results to peers’.	a) T hands out the worksheets for the second problem. b) Ss are asked to form different pairs and work on this second task with the new classmate. c) In this case Ss do not simply have to apply a known procedure, but they need to explore three new functions in GeoGebra in order to get to the solution. d) The last question will help Ss to think critically and to consider a problem in a different (and easier) way. e) T circulates to facilitate. f) After 15 minutes, T asks Ss to share their results in plenary.	**Skills** L S R W **Key vocabulary** upper/lower sum, area, approximation, rectangles, round down, round up, decimal digit **Communicative structures** “This function simplifies our work a lot!” “We can change the number of subdivisions …” “Do you think there is another way to…?”	☑ Whole class ☐ Group work ☑ Pair work ☐ Individual work	• U1_L3_ALL3.zip • U1_L3_ALL2.pdf Ss should create, as a solution for the task in U1_L3_ALL2, a file similar to U1_L3_ALL3.	Formative: T models language and cognition. Peer and self-assessment.

4	25'	a) Applying a learnt procedure to a new situation (a new function). b) Using GeoGebra as a tool to solve a problem. c) Learning from peers/teaching to peers (peer-to-peer). d) Cooperation and communication. e) Comparing own results to peers'.	a) T hands out the worksheets for the third (and last) problem. b) Ss are asked to work on their task in pairs (the same pairs as for activity 2). c) The reason why they are asked to work in the original pairs is that they can learn from each other, after having worked with a different person (during activity 3). d) T circulates, observes and listens to Ss. e) After 15 minutes, T asks Ss to share their results in plenary. f) T sums up the main points and ideas of this lesson and underlines again the relations between the integral of a function and the area under the graph of the function. g) T gives Ss some more exercises to consolidate what they have learnt.	**Skills** L S R W **Key vocabulary** upper/lower sum, area, approximation, rectangles, round down, round up, decimal digit **Communicative structures** "We could proceed in a similar way…" "How can we change the number of decimal digits in GeoGebra…?" "Before we did…, now we could…" "We have seen that the greater the number of subdivisions n, the better the approximation…"	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	• U1_L3_ALL4.pdf • U1_L3_ALL5.zip Ss should create, as a solution, a file similar to U1_L3_ALL5. Exercises to consolidate what has been learnt about the relation between integral and area.	Formative: T elicits language and cognition. Peer and self-assessment.

CLIL Lesson Plan

Unit number	1	**Lesson number**	4	**Title**	Integrals – properties

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**
1	5'	a) Reviewing the concepts and definitions examined in the previous lesson. b) Consolidating the knowledge acquired in the previous lesson. c) Knowing the aim of the lesson.	a) T briefly reviews the concepts and the definitions studied in the previous lesson. b) Ss can ask questions and clarifications. c) T sets the goals for the lesson.	**Skills** L S R W **Key vocabulary** Integral, function, area **Communicative structures** “In the previous lessons we learnt...” “Today we’re going to study...” “Excuse me, could you please explain why...”	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work		

2	15'	a) Employing the knowledge acquired during the previous lessons. b) Re-elaborating autonomously the concepts learnt previously. c) Making hypotheses. d) Sharing ideas and hypotheses with peers. e) Critical thinking. f) Giving comments/opinions. g) Cooperating.	a) T asks the Ss to make hypotheses about the result of the integral of a function multiplied by a constant and of the sum/difference of two functions. b) Ss think about the answer independently for 5 minutes. c) After 5 minutes expire Ss form pairs and discuss the answer with their pair-mate. d) After 5 more minutes the discussion is extended to the whole class.	**Skills** L S R W **Key vocabulary** Integral, function, area, constant, sum, difference **Communicative structures** “If we multiply the function times a constant, then the area...” “If we sum two functions then...” “Do you think that this hypothesis is correct?”	☑ Whole class ☐ Group work ☑ Pair work ☑ Individual work	T writes the main concepts and ideas emerged during the discussion on the blackboard. Ss exploit the calculations in their personal notebooks.	During the activity T goes around the class assessing the level of participation and comprehension of the Ss. Self-assessment: Ss compare their version of the solution to that found by their pair-mate.

3	20'	a) Learning how to solve the integral of a function multiplied by a constant and of the sum/difference of two functions. b) Identifying relevant concepts and information. c) Practicing listening and reading skills.	a) T shows to the class two videos from Khan academy about the integral of a scaled function and about the integral of the sum of two functions. b) Ss take notes and can ask questions and clarifications. c) T pauses the videos to answer questions and give clarifications.	**Skills** L S R W **Key vocabulary** integral, function, scaled, area, constant, sum, difference **Communicative structures** “Could you pause the video?” “Could you explain what... means?” “Could you clarify the passage about ...?”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T runs the video “Integrating scaled version of a function” (link:link) on the IWB. T runs the video “Integrating sums of functions” (link: link) on the IWB. Ss take notes in their personal notebooks.	

4	20'	a) Exploiting the knowledge acquired during the previous activities. b) Making hypotheses. c) Using the software GeoGebra to prove/disprove the validity of the hypotheses made. d) Cooperating. e) Giving comments/opinions.	a) T asks Ss to work in pairs on the exercise sheet “Integrals – Quiz” (file U1_L4_ALL1.pdf). b) Ss, working in pairs, formulate hypotheses to answer the requests. c) After 8 minutes expire T asks the students to use GeoGebra to prove the validity of their hypotheses. d) After 8 more minutes the solutions are discussed in plenary.	**Skills** L S R W **Key vocabulary** integral, function, scaled, area, constant, sum, difference, absolute value, greater than (or equal to), less than (or equal to), integration boundaries **Communicative structures** “Complete the following identity…” “Choose one of the options…”	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	• U1_L4_ALL1.pdf • U1_L4_ALL2.zip T hands out to each pair of Ss a printed copy of the exercise sheet “Integrals – Applications in physics” (file U1_L4_ALL1.pdf – editable version: U1_L4_ALL2.zip).	During the activity T goes around the class assessing the level of comprehension and participation of the Ss. Self-assessment: Ss compare their solution to the correct one.

5	10'	a) Re-elaborating in a more formal way the concepts encountered during the previous activities. b) Learning the main properties of integrals. c) Taking notes.	a) T summarises the properties of integrals encountered in the previous activities. b) Ss take notes and ask questions and clarifications.	**Skills** L S R W **Key vocabulary** integral, function, scaled, area, constant, sum, difference **Communicative structures** “The integral of the sum of two functions is equal to the sum of the integrals of the single functions” “The integral of a function multiplied by a constant is...”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T writes the main concepts on the blackboard. Ss take notes in their personal notebooks.	

6	10'	a) Exploiting the knowledge acquired during the previous actitvities. b) Interpreting graphs. c) Practicing problem solving skills. d) Understanding written instructions. e) Self-assessment.	a) T gives Ss instructions to reach the web-pages “Definite integral by thinking about the function's graph” and “Using multiple properties of definite integrals” from Khan Academy. b) Ss follow the instructions and solve the exercises. c) Ss receive automatically feedback about their solutions.	**Skills** L S **R** **W** **Key vocabulary** integral, function, scaled, area, constant, sum, difference, graph **Communicative structures** “Stuck?” “Use a hint”	☐ Whole class ☐ Group work ☐ Pair work ☒ Individual work	Ss use the PCs available in the ICT lab. T gives Ss instructions to reach the web-pages relative to the two online activities. Ss follow the links link (for the activity “Definite integral by thinking about the function's graph”) and link (for the activity “Using multiple properties of definite integrals”).	During the activity T goes around the class assessing the level of comprehension of Ss. Self-assessment: Ss receive immediate feedback on their answers.

7	20'	a) Exploiting the knowledge acquired during the previous activities and lessons to solve two exercises about the calculation of an integral. b) Cooperating. c) Giving comments/opinions.	a) T asks Ss to work in pairs on the solution of the exercise from the exercise sheet “Integrals – properties” (file U1_L4_ALL3.pdf). b) Ss work in pairs to solve the exercise. c) After 10 minutes expire each pair of Ss compares the solution with another pair.	**Skills** L S R W **Key vocabulary** integral, function, properties **Communicative structures** “Solve the following integral using...”	☐ Whole class ☐ Group work ☑ Pair work ☐ Individual work	• U1_L4_ALL3.pdf • U1_L4_ALL4.zip T hands out to each pair of Ss a printed copy of the exercise sheet “Integrals – properties” (file U1_L4_ALL3.pdf – editable version: U1_L4_ALL4.zip).	During the activity T goes around the class assessing the level of comprehension and participation of the Ss. Self-assessment: Ss compare their solutions to that of another pair.

CLIL Lesson Plan

Unit number	1	**Lesson number**	5	**Title**	The integral function

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**
1	5'	a) Revising the properties of integrals. b) Consolidating the knowledge acquired in the previous lesson. c) Knowing the aim of the lesson.	a) T asks Ss to briefly revise the properties of the integrals they studied in the previous lesson. b) Ss ask for clarifications if they still have any doubts. c) T introduces objective and learning outcomes of this lesson.	**Skills** L S R W **Key vocabulary** integral, function, scaled, area, constant, sum, difference, absolute value, greater than (or equal to), less than (or equal to), integration boundaries **Communicative structures** “Today we’re going to talk about…” “Could you please tell the class one of the properties of the integrals?” “The integral of a sum/difference of functions…”	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work		

2	15'	a) Understanding the concept of integral function. b) Taking notes. c) Communicatin	a) T defines a new mathematical object that is related to the calculus of the area under the graphs of a function within a specific interval: the integral function. b) T clarifies the new concepts providing some examples on the blackboard. c) Ss take notes and ask for clarifications if needed.	**Skills** L S R W **Key vocabulary** integral function, interval, area **Communicative structures** “Consider a function f defined over an interval [a;b]. We define the integral function of f as the function that associates to each x in [a;b] the value of the integral…” “Excuse me, could you please repeat…?”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T writes the definition of integral function and some examples on the blackboard. Ss take notes in their personal notebooks.	
3	25'	a) Applying a definition to new particular situations. b) Interpreting graphs. c) Practicing problem solving skills. d) Understanding written instructions. e) Communicating.	a) T hands out the worksheets and asks Ss to solve the exercises in pairs. b) Ss work in pairs and ask for clarification if needed. c) T circulates to facilitate. d) Then T asks some Ss to solve the problems on the blackboard, so that every S can check them. e) These Ss	**Skills** L S R W **Key vocabulary** graph, function, integral, integral function, polynomial	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	• U1_L5_ALL1.pdf • U1_L5_ALL2.zip The editable version of U1_L5_ALL1 is: U1_L5_ALL2.	Formative: T models language, cognition and content. Peer and self-assessment.

		f) Observing and recognising analogies between different polynomial functions and their integral function. g) Making hypothesis about the degree of the integral function of a polynomial.	correct the problems on the blackboard and the solutions are discussed in plenary. f) Exercises n° 1, 3 and 4 are particularly interesting because here Ss can find the expression of the integral function of a given function. Since the three given functions and their respective integral functions are all polynomials, T asks Ss to compare the degree of each function with the degree of its integral function. g) Ss can make hypotheses to generalize the rule they have just noticed.	**Communicative structures** “In order to determine the integral function of this function we can simply consider that the area of the rectangle of length x is…” “I’m not sure because…” “The integral function of a first degree polynomial is a second degree polynomial.” “Do you think that this could be a general result? Why?”			

4	15'	a) Practicing using GeoGebra to visualise the integral function of a given function. b) Understanding how to visualise the integral function of a function using GeoGebra. c) Interpreting graphs. d) Practicing listening skills. e) Making hypotheses and justifying them. f) Generalising results.	a) In order to explore this issue more deeply, T guides Ss to create their GeoGebra file U1_L5_ALL3. b) Ss use the PCs available in the ICT lab and work in pairs. They follow the instructions of T and create the GeoGebra file “Integral function” (see U1_L5_ALL3). c) When all the pairs have completed the task, T asks them to discuss the result. d) T asks Ss to suggest hypotheses for the integral function of a fourth degree polynomial.	**Skills** L S R W **Key vocabulary** graph, integral, function, integral function, area, locus **Communicative structures** “Let’s define a locus…” “What did you get as integral function of our third degree polynomial?” “Is this result consistent with the hypotheses you formulated before?” “What if the original function was a fourth degree polynomial?” “If the function was a fourth degree polynomial… would be…” (And similar second conditional structures.)	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	• U1_L5_ALL3.zip Ss use the PCs available in the ICT lab. T creates the file U1_L5_ALL3 together with Ss, giving them instructions. As they are creating it, the file U1_L5_ALL3 is shown on the whiteboard.	

5	20'	a) Exploring an issue observing particular cases and interpreting them. b) Making hypotheses about the form of the integral function of some functions. c) Critical thinking. d) Giving opinions. e) Communication and discussion.	a) T asks Ss to work in pairs and create similar GeoGebra files (see U1_L5_ALL4), considering other functions: at least three other polynomials of different degrees that they can choose autonomously and the following other functions: f(x)=e^x (with integration boundaries 1 and x), f(x)=cos(x), f(x)=sin(x) (with integration boundaries –π/2 and x), f(x)=1/x (with x>0 and with integration boundaries 1 and x). b) Ss are asked to recognise the integral function of each function and write them in their personal notebook in a table (see U1_L5_ALL5 and U1_L5_ALL6 for the solution). c) T circulates to facilitate. d) After 15 minutes, T asks Ss to compare and discuss the results with the entire class.	**Skills** L S R W **Key vocabulary** polynomial, degree, integral function, exponential, sine, cosine, natural logarithm **Communicative structures** “This integral function seems to be f(x)=x^3, do you agree?” “Let’s check it by drawing the graph of f(x)=x^3 and comparing it with the graph of the integral function” “What happens if we take x<0?” “The problem is that in x=0 the function f(x)=1/x is not defined!” “I’m quite sure that this is the graph of the natural logarithm.”	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	• U1_L5_ALL4.zip • U1_L5_ALL5.pdf • U1_L5_ALL6.pdf An example of the files Ss are asked to produce is U1_L5_ALL4. U1_L5_ALL5 for the activity “Recognise the integral function” (see U1_L5_ALL6 for the solution).	Formative: T circulates and models language and cognition. Peer and self-assessment.

6	20'	a) Consolidating knowledge about the integral function. b) Using the acquired knowledge to solve new problems. c) Improving problem solving skills. d) Communicating.	a) T hands out the worksheets for the last activity. b) Ss are asked to complete the task on the worksheet individually. c) T circulates, observes and gives a hint where needed. d) After 15 minutes, T asks Ss to share their results in plenary and corrects the problems. e) T gives Ss similar exercises for homework.	**Skills** L S R W **Key vocabulary** integral function, graph, integration boundaries **Communicative structures** “I think that the second graph represents the integral function of f because…” “I’m pretty sure that the third graph can’t represent the integral function of f because…	☒ Whole class ☐ Group work ☐ Pair work ☒ Individual work	• U1_L5_ALL7.pdf • U1_L5_ALL8.zip U1_L5_ALL8 is the editable version of U1_L5_ALL7.	Formative: T elicits language, concepts and cognition. Self-assessment.

CLIL Lesson Plan

Unit number	1	**Lesson number**	6	**Title**	Integrals – the fundamental theorem of calculus

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**
1	15'	a) Using the knowledge acquired during the previous lessons to solve an exercise about integral functions. b) Interpreting a graph. c) Cooperating. d) Giving comments/opinions.	a) T asks Ss to work in pairs on the solution of the exercise from the exercise sheet “Integrals – Integral function – part 1” (file U1_L6_ALL1.pdf). b) Ss work in pairs to solve the exercise. c) After 10 minutes expire the solution is discussed with the entire class.	**Skills** L S R W **Key vocabulary** integral, function, graph, options **Communicative structures** “Which of the following...is...?”	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	• U1_L6_ALL1.pdf • U1_L6_ALL2.zip T hands out to each pair of Ss a printed copy of the exercise sheet “Integrals – Integral function – part 1” (file U1_L6_ALL1.pdf – editable version: U1_L6_ALL2.zip).	During the activity T goes around the class assessing the level of comprehension and participation of the Ss. Self-assessment: Ss compare their solution to the correct one.

2	5'	a) Reviewing the concepts and definitions examined in the previous lesson. b) Consolidating the knowledge acquired in the previous lesson. c) Knowing the aim of the lesson.	a) T briefly reviews the concepts and the definitions studied in the previous lesson. b) Ss can ask questions and clarifications. c) T sets the goals for the lesson: learning the fundamental theorem of calculus and its basic applications.	**Skills** L S R W **Key vocabulary** integral, function, fundamental, theorem, calculus, graph **Communicative structures** “The aim of the lesson is...” “We are going to learn...”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work		

3	30'	a) Learning the fundamental theorem of calculus. b) Following a theoretical explanation. c) Identifying important information and concepts. d) Taking notes.	a) T explains the fundamental theorem of calculus and proves it. The theoretical explanation is based on the document U1_L6_ALL3.doc. c) Then T shows and proves how to calculate a definite integral and provides some examples. d) Ss take notes and can ask questions and clarifications.	**Skills** L S R W **Key vocabulary** integral, function, fundamental, theorem, calculus, boundaries, variable, graph **Communicative structures** “The fundamental theorem of calculus states that...” “The integration boundaries are...”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L6_ALL3.pdf The theoretical explanation is based on the document U1_L6_ALL3.pdf.	

4	15'	a) Employing the knowledge acquired during the previous activities. b) Practicing problem solving skills. c) Understanding written instructions. d) Self-assessment.	a) T gives Ss instructions to reach the web-page “Calculating derivatives using the fundamental theorem of calculus” from Khan Academy. b) Ss follow the instructions and solve the exercises. c) Ss receive automatically feedback about their solutions.	**Skills** L S **R** **W** **Key vocabulary** integral, function, fundamental, theorem, calculus, boundaries, variable, graph, options **Communicative structures** “Stuck?/Use a hint.”	☐ Whole class ☐ Group work ☐ Pair work ☒ Individual work	Ss use the PCs available in the ICT lab. T gives Ss instructions to reach the web-page “Calculating derivatives using the fundamental theorem of calculus”. Ss have to follow the link link.	During the activity T goes around the class assessing the level of comprehension of Ss. Self-assessment: Ss receive immediate feedback on their answers.

5	20'	a) Learning how to calculate the indefinite integral of basic functions. b) Making links between new concepts and prior knowledge. d) Following a theoretical explanation. e) Identifying important information and concepts. f) Taking notes.	a) T asks Ss how to calculate the definite integrals of basic function (e.g. powers, sine, cosine, exponential, logarithm,...) using the result they have just proven and draws a table on the blackboard. b) The results are compared to the results found during the activity “Recognise the integral function” of lesson 5. c) Ss take notes and can ask questions and clarifications.	**Skills** L S R W **Key vocabulary** fundamental, theorem, calculus, sine, cosine, power, exponential, logarithm **Communicative structures** Sentence structures related to mathematical functions. “Indefinite integral of sin(x) is...”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T writes the main results on the blackboard. Ss take notes in their personal notebooks.	During the part of the activity dedicated to the comparison of the results of those of the activity “Recognise the function” T asssesses the content of the contributions by the Ss as well as the language used.

6	15'	a) Employing the knowledge acquired during the previous activities. b) Practicing the calculation of indefinite integrals. c) Improving problem solving skills.	a) T writes at the blackboard a definite integral. b) Ss work individually on the solution. c) The first S to solve the integral gives the solution. d) T confirms/corrects the solution. e) The first S to solve correctly the integral gets 2 points. f) The activity is repeated until the time of the lesson expires. g) One of the exercises is used to introduce the mean value theorem. h) The S with more points at the end of the activity receives a symbolic reward.	**Skills** L S R W **Key vocabulary** integral, power, sine, cosine, power, exponential, logarithm **Communicative structures** Sentence structures related to mathematical functions. “Ready? Set? Go!” “Done!” “My result is...” “The result is correct/incorrect”	☐ Whole class ☐ Group work ☐ Pair work ☒ Individual work	T writes the integrals on the blackboard. Ss solve the exercises on their personal notebooks.	T assesses the problem solving skills of Ss and the language they use for their contributions.

CLIL Lesson Plan

Unit number	1	**Lesson number**	7	**Title**	Methods of integration

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**
1	10'	a) Revising acquired knowledge (fundamental theorem of calculus and its applications). b) Consolidating acquired knowledge (fundamental theorem of calculus and its applications). c) Understanding the objective of the lesson.	a) T shows some exercises on the IWB (see U1_L7_ALL1) and asks the Ss to complete them individually as a revision of the calculus of definite integrals introduced in the last lesson. b) When Ss have finished, T collects the worksheets. c) Ss can ask questions if they have any. d) T briefly introduces objective and learning outcomes of this lesson.	**Skills** L S R W **Key vocabulary** integral function, area, graph, polynomial, sine, cosine, natural logarithm **Communicative structures** “Excuse me, I didn’t understand… very well.” “Today we’re going to see two interesting methods for… and…”	☑ Whole class ☐ Group work ☐ Pair work ☑ Individual work	• U1_L7_ALL1.pdf T shows U1_L7_ALL1 on IWB. Ss calculate the definite integrals on a sheet of paper that T gives them.	Formative: T collects the revisions and assesses them.

2	20'	a) Recognising integrals of derivatives of compositions of two functions. b) Understanding how the chain rule works with integrals. c) Improving problem solving skills. d) Cooperating. e) Creative thinking. f) Communicating.	a) T uses the last exercise of the first activity for introducing a method of integrating functions based on the chain rule for calculating the derivative of a composition of functions. b) T forms the groups and hands out the worksheets for the activity (see U1_L7_ALL2). c) Ss work in groups on their task. d) T circulates, observes, listens to Ss and clarifies some interesting points if necessary.	**Skills** L S R W **Key vocabulary** chain rule, derivative, internal/external function **Communicative structures** “I think f(x) is…” “This is the derivative of the function…” “Are you sure about these two functions?” “What if you multiply and divide by a number in order to get the derivative of the internal function?"	☐ Whole class ☒ Group work ☐ Pair work ☐ Individual work	• U1_L7_ALL2.pdf	T models language, cognition and concepts.

3	20'	a) Consolidating knowledge and formalising. b) Understanding the substitution method for solving integrals. c) Listening to others. d) Taking notes. e) Communication and discussion.	a) T asks Ss to share their results with the rest of the class. b) Ss discuss these results, T writes on the blackboard the solutions and Ss check them. c) Now T explains the substitution method providing some examples on the blackboard. T tries to make Ss interact as much as possible. d) Ss interact with T, ask questions if necessary and take notes.	**Skills** L S R W **Key vocabulary** substitution, auxiliary variable **Communicative structures** “If we call 2x …” “Excuse me, how can I decide which substitution is the best?” “If you have the same term more than once, then you could… or if there is a term that… ”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T checks the solutions of the previous exercises and writes some examples of integration by substitution on the blackboard, while Ss take notes in their personal notebooks.	

4	15'	a) Putting into practice what T has just explained (substitution method) in new situations. b) Understanding and consolidating knowledge. c) Improving one’s own problem solving skills. d) Cooperating.	a) T hands out the worksheets for the next activity (see U1_L7_ALL3). b) Ss work on their task in pairs and ask questions if necessary. c) T circulates to facilitate. After 15 minutes, T asks Ss to share their results in plenary and writes the solutions on the blackboard.	**Skills** L S R W **Key vocabulary** substitution, auxiliary variable **Communicative structures** “In my opinion here we should substitute…” “I don’t agree. I think it’s easier if we…”	☐ Whole class ☐ Group work ▣ Pair work ☐ Individual work	• U1_L7_ALL3.pdf	Formative: T models language, content and cognition.

5	20'	a) Finding a formula to calculate integrals of a product of two functions. b) Understanding when and learning how to calculate integrals by parts. c) Applying a new formula to calculate a specific integral. d) Understanding a written text. e) Cooperation and communication. f) Listening to others.	a) T briefly introduces the activity, hands out the worksheets (U1_L7_ALL4) and asks Ss to complete them in pairs. b) Ss work in pairs and ask questions if necessary. c) When Ss have finished, they compare their result with that of another pair of Ss next to them. d) T circulates and clarifies if needed. e) After 15 minutes, T discusses with Ss and formalises the method of integration by parts on the blackboard.	**Skills** L S R W **Key vocabulary** integration by parts, derivative, product **Communicative structures** “If we choose … as f(x), then we will be stuck, because… whereas if we…” “I perfectly agree with you!” “I’m not so sure about that because if you consider…” “Let’s ask them if they got the same result as us.”	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	• U1_L7_ALL4.pdf	T models cognition and concepts. Peer-assessment.

6	15'	a) Using the knowledge acquired during the previous activity. b) Applying the integration method by parts to new problems. c) Practicing problem solving skills. d) Cooperating. e) Self-assessment	a) T gives Ss instructions to reach the web-page “Integration by parts: definite integrals” from Khan Academy. b) Ss follow the instructions and solve the exercises in pairs. c) Ss automatically receive feedback about their solutions. d) T gives Ss some exercises for homework about all the different methods of integration studied during this lesson.	**Skills** L S R W **Key vocabulary** integration by parts, evaluate, hint **Communicative structures** “I’m sure this is correct!” “Use a hint/get help/move on”	☐ Whole class ☐ Group work ☑ Pair work ☐ Individual work	Ss use the PCs available in the ICT lab. T gives Ss instructions to reach the website for the activity “Integration by parts: definite integrals”. Ss follow the link link.	Formative: during the activity T goes around the class assessing the level of comprehension of Ss. Self-assessment: Ss receive immediate feedback on their answers.

CLIL Lesson Plan

Unit number	1	**Lesson number**	8	**Title**	Integrals – Problem solving challenge

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**
1	10'	Understanding the text and the requests of a “complex” problem.	a) T introduces objective, learning outcomes and activities of this lesson. b) Ss are going to deal with a “complex” problem: problem number 2 from the state exam of 2016 for Italian scientific high schools. c) T forms the groups and hands out the text of the problem. d) T asks a S to read the text aloud and clarifies the meaning of new vocabulary and, if necessary, the requests of the problem.	**Skills** L S R W **Key vocabulary** graph, derivative, derivable, tangent, maximum, minimum, inflection point/flex, half-line **Communicative structures** “Today you’re going to work in groups on a “complex” problem…” “Could you please read the text?” “Please, ask questions if you do not know the meaning of a word.”	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L8_ALL1.pdf • U1_L8_ALL2.zip The text of the problem can be found in file U1_L8_ALL1.pdf (editable version file U1_L8_ALL2.zip).	

2	55'	a) Applying acquired knowledge to a complex problem. b) Problem solving. c) Creative thinking: Ss have to use all the mathematical tools at their disposal to solve the problem. d) Making connections. e) Organising the work in a group. f) Time managing. g) Cooperating.	a) Ss work in groups. b) Ss can organise the work as they prefer (working together or assigning different tasks to each member of the group according to their abilities and then sharing results). c) T supervises the work asking targeted questions.	**Skills** L S R W **Key vocabulary** graph, derivative, derivable, tangent, maximum, minimum, inflection point/flex, half-line **Communicative structures** “How can we organise the work?” “I could work on...” “Who is going to draw the graphs?” “The derivative of f evaluated in x=3 results...” “The integral results...”	☐ Whole class ☒ Group work ☐ Pair work ☐ Individual work	• U1_L8_ALL1.pdf	Formative: T assesses the level of participation and Ss in the group work. T models and elicits language, cognition and concepts.

3	15'	a) Analysing peers' work/Peer assessment. b) Critical thinking. c) Learning from the work of others.	a) Ss are asked to exchange their work (group 1 gives theirs to group 2, group 2 gives theirs it to group 3,…). b) T asks Ss to analyse the work of the other group, compare it with their own, in order to formulate some suggestions or corrections to improve it. c) This work is useful also to understand if the own work is correct and/or if it is possible to improve it. d) T circulates and observes the work of Ss.	**Skills** L S R W **Key vocabulary** graph, derivative, derivable, tangent, maximum, minimum, inflection point/flex, half-line **Communicative structures** "Look at their solution! Their … is different from ours: let's try to understand who's right!" "We forgot to consider…" "Good, their result is the same as ours!"	☐ Whole class ☒ Group work ☐ Pair work ☐ Individual work	• U1_L8_ALL1.pdf	Formative: T models and elicits language, cognition and concepts. Peer and self-assessment.

4	10'	a) Critically analysing peers' correction. b) Listening to others.	a) Ss return the work to the original groups. b) One person per group leaves their group in order to explain the corrections suggested during the previous activity to the other group. c) Ss analyse critically the corrections/suggestions that have been made by the other group. d) T supervises and listens to Ss.	**Skills** L S R W **Key vocabulary** graph, derivative, derivable, tangent, maximum, minimum, inflection point/flex, half-line **Communicative structures** "We notice that you forgot to consider…" "We think that in order to find… you need to…" "I think they're right."	☐ Whole class ☒ Group work ☐ Pair work ☐ Individual work	• U1_L8_ALL1.pdf	Formative: T models and elicits language, cognition and concepts. Peer and self-assessment.

5	10'	a) Evaluating peers' and own work. b) Reflecting on different solution strategies. c) Reflecting on the importance of group work. d) Understanding what a state exam problem is like and understanding the importance of considering all the available mathematical tools in order to find a solution.	a) T asks Ss to share their results in plenary. b) In case of doubts, T corrects the problem. c) T invites Ss to reflect on how, in the state exams, all the mathematical knowledge acquired during the 5 years can be necessary. d) T asks Ss to comment on the activity (did they find it useful to exchange the work with another group? Why?...)	**Skills** L S R W **Key vocabulary** graph, derivative, derivable, tangent, maximum, minimum, inflection point/flex, half-line **Communicative structures** "In order to sketch the graph of F(x) you need to…" "As you notice, here you needed to use the mean value theorem." "To solve part 4 you have to…" "I think that comparing our results was useful because..."	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T writes at the blackboard the necessary corrections. Ss take notes.	Peer and self-assessment.

CLIL Lesson Plan

Unit number	1	**Lesson number**	9	**Title**	Integrals – physics applications + exercises in preparation to the unit test

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**
1	5'	Knowing the aim of the lesson.	a) T sets the goals for the lesson: understanding the importance of integrals in physics and practicing the problem solving skills in preparation to the unit test. b) Ss can ask questions and clarifications.	**Skills** L S R W **Key vocabulary** integral, quantity, space, time, velocity, acceleration, work, force, displacement, charge, current **Communicative structures** “During this lesson we are going to discuss some of the main applications of integrals in physics.” “You are going to solve a few exercises in preparation to the unit test of next lesson.”	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work		

2	20'	a) Learning the main applications of integrals in physics (in kinematics, in dynamics, and in electrodynamics), b) Following a theoretical explanation. c) Making links between different subjects. d) Identifying important information and concepts. e) Taking notes.	a) T invites Ss to reason on the relation between the position of a particle and its velocity starting from a velocity vs. time graph. T conducts the discussion to the point where Ss realise that the displacement of a particle can be calculated as the definite integral of the function v(t) that describes the velocity of the particle as a function of time. b) The same procedure is followed for the relation between the work done by a force and the displacement produced by said force, and the relation between charge and current. c) Ss participate actively to the discussion. d) Ss	**Skills** L S R W **Key vocabulary** Integral, quantity, space, time, velocity, acceleration, work, force, displacement, charge, current **Communicative structures** “How can we use this v vs. t graph to calculate the displacement of the particle?” “How can we generalise the procedure of calculating the area under the curve representing v(t)?” “What is the definition of the work done by a force?” “What is the definition of current (intensity)?”	☒ Whole class ☐ Group work ☐ Pair work ☐ Individual work	T writes at the blackboard the main points of the discussion (and sketches the necessary graphs).	

			take notes and can ask questions and clarifications.				
3	25'	a) Employing the knowledge acquired during the previous activities. b) Combining knowledge and methods from different subjects. c) Practicing problem solving skills. d) Understanding written instructions. e) Cooperating. f) Self- and peer-assessment.	a) T gives Ss instructions to reach the web-pages “Analyzing motion problems (integral calculus)” and “Motion problems (with integrals)” from Khan Academy. b) Ss follow the instructions and solve the exercises individually. c) Ss receive automatically feedback about their solutions.	**Skills** L S **R** W **Key vocabulary** Integral, motion, space, time, velocity, position, instant **Communicative structures** “The velocity of a particle moving along the x-axis is...” “What is the position of the particle...?”	☐ Whole class ☐ Group work ☐ Pair work ▣ Individual work	Ss use the PCs available in the ICT lab. T gives Ss instructions to reach the website for the activities. Ss follow the links link (for the activity “Analyzing motion problems (integral calculus)”) and link (for the activity “Motion problems (with integrals)”).	During the activity T goes around the class assessing the level of comprehension of the Ss. Self-assessment: Ss receive immediate feedback on their answers.

4	20'	a) Exploiting the knowledge acquired during the previous lessons to solve three exercises about the applications of integrals in physics. b) Calculating definite integrals. c) Combining knowledge and methods from different subjects. d) Cooperating. e) Giving comments/opinions.	a) T asks Ss to work in pairs on the solution of exercises 1, 3, 4 from the exercise sheet “Integrals – Applications in physics” (file U1_L9_ALL1.pdf). b) Ss work in pairs to solve the exercises. c) After 15 minutes expire the solution is discussed with the entire class. d) The solution of exercise 2 is left as a homework in preparation to the unit test.	**Skills** L S R W **Key vocabulary** Integral, motion, space, time, mass point, velocity, position, instant, work, force, charge, current, circuit **Communicative structures** “The velocity of a particle at a given time is described by...” “A mass point moves on a straight line with velocity...” “Calculate the amount of charge that flows through…”	☒ Whole class ☐ Group work ☒ Pair work ☐ Individual work	• U1_L9_ALL1.pdf • U1_L9_ALL2.zip T hands out to each pair of Ss a printed copy of the exercise sheet “Integrals – Applications in physics” (file U1_L9_ALL1.pdf – editable version: U1_L9_ALL2.zip).	During the activity T goes around the class assessing the level of comprehension and participation of the Ss. Self-assessment: Ss compare their solution to the correct one.

5	30'	a) Exploiting the knowledge acquired during the previous lessons to solve three exercises that cover the main topics encountered during the present CLIL module. b) Calculating definite integrals. c) Combining knowledge and methods from different subjects. d) Time management. e) Giving comments/opinions.	a) T asks Ss to work individually on the solution of exercises from the exercise sheet “Integrals – Exercises in preparation to the unit test” (file U1_L9_ALL3.pdf). b) T gives the start for the solution of each exercise. c) The first S to solve the integral gives the solution. d) T confirms/corrects the solution. e) The first S to solve correctly the integral gets 2 points. f) The activity is repeated until the time of the lesson expires. g) The S with more points at the end of the activity receives a symbolic reward.	**Skills** L S R W **Key vocabulary** integral, sine, cosine, power, option, integration by parts, integration by substitution, integration boundaries, continuous, half-circle **Communicative structures** Sentence structures related to mathematical functions. “The graph of … consists of three half-circles centered in...” “Ready? Set? Go!” “Done!” “My result is...” “The result is correct/incorrect.”	☒ Whole class ☐ Group work ☐ Pair work ☒ Individual work	• U1_L9_ALL3.pdf • U1_L9_ALL4.zip T hands out to each pair of Ss a printed copy of the exercise sheet “Integrals – Exercises in preparation to the unit test” (file U1_L9_ALL3.pdf – editable version: U1_L9_ALL4.zip).	During the activity T goes around the class assessing the level of comprehension and participation of the Ss. Self-assessment: Ss compare their solution to the correct one.

CLIL Lesson Plan

Unit number	1	**Lesson number**	10	**Title**	Unit test

Activity	**Timing**	**Learning Outcomes**	**Activity Procedure**	**Language**	**Interaction**	**Materials**	**Assessment**
1	60'	a) Applying acquired knowledge to new problems and situations. b) Using appropriate terminology. c) Analyzing new problems and reasoning critically.	a) T reads the questions of the unit test and asks if everything is clear. b) Ss complete the test individually.	**Skills** L S **R** **W** **Key vocabulary** real-valued function, odd function, velocity, acceleration, to decelerate **Communicative structures** “Tell me if you do not understand the text.”	☐ Whole class ☐ Group work ☐ Pair work ☒ Individual work	• U1_L10_ALL1.pdf	Summative: T collects the tests and assesses them.

2	25'	Evaluating own work.	a) T discusses and corrects the exercises assigned in the unit test. b) Ss take notes and ask for clarifications where needed.	**Skills** L S R W **Key vocabulary** real-valued function, odd function, velocity, acceleration, to decelerate **Communicative structures** “For this exercise you could simply…” “Is it correct if I…?” “The easiest way is…” “It’s not important that the functions are not completely positive in the considered interval because…”	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work	• U1_L10_ALL1.pdf T corrects the exercises assigned in the unit test on the blackboard. Ss take notes on their personal notebooks.	Peer and self-assessment.
3	15'	a) Evaluating this CLIL Module. b) Being aware of personal improvement (mathematical, linguistic and social).	a) T discusses this CLIL experience with Ss. b) Ss are invited to express their opinions, ideas, comments about all the aspects of this experience: the topic, the teaching methods, how they evaluate the learning outcomes (did they learn something more or different in comparison with other	**Skills** L S R W **Key vocabulary** Group work, TPS, individual work, learning outcomes, skills, life skills, social skills	☑ Whole class ☐ Group work ☐ Pair work ☐ Individual work		

			comparison with other Modules in Italian?), if they found it difficult to understand/speak in English, if they enjoyed the activities, which activity they enjoyed the most, suggestions to improve the CLIL experience for other classes,…	**Communicative structures** • “Give me some overall feedback about this experience.” • “Which activity did you enjoy the most?” • “The activity… helped me the most to understand the fundamental theorem of calculus…” • “Do you think you practiced some important skills that you don’t usually practice at school during “normal” lessons?” • “I found it easy to understand the lessons, whereas speaking was more difficult.” • “I learnt how to deal with new apparently difficult problems…” • “I’d have liked to do more…” • “In my opinion working in pairs was great and useful because…"			